

Tomography fundamentals

Kip Matthews
Medical Physics and Health Physics Program
Dept. of Physics & Astronomy

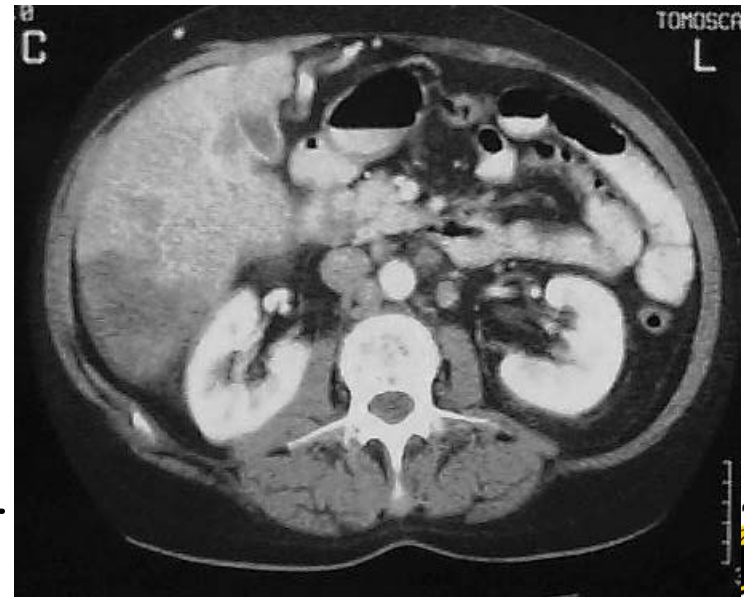


Definitions

- Conventional x-ray images
 - a 2D projection of a 3D object
 - e.g., a chest x-ray, “shadows” of internal anatomy
- X-ray tomography, tomographic images
 - a stack of 2D cross-sectional images of a 3D object



Chest x-ray



Abdominal CT

Radiation sources for tomography

- Several types of radiation can be used for tomography
 - Neutrons
 - Gamma rays
 - X-rays
 - Potentially others (radiofrequency, ultrasound, ...)
- X-rays are probably the most common radiation source for tomography
 - X-ray tomography is used in
 - Medicine
 - Industry
 - Research

Facts about x-rays

- X-rays are a form of photon radiation
 - X-rays are a form of electromagnetic radiation
 - Very short wavelengths compared to visible light
 - Consequently much higher in energy than visible light
- X-rays were discovered in December 1895 by Wilhelm Roentgen
 - X-rays were rapidly implemented for medical applications
 - In January 1896, physicians used an x-ray image to guide the extraction of a metal shard from a worker's hand
 - X-rays were also quickly adopted for industrial and research use

More facts about x-rays

- X-rays are “penetrating” radiation
 - X-rays can travel “long” distances through matter
 - “long” depends on the x-ray’s energy and the type of matter
- X-rays are ionizing radiation
 - An x-ray carries sufficient energy to ionize an atom when absorbed
 - The x-ray energy is converted into kinetic energy of the ionized electron
 - which can deposit this energy into surrounding matter through collisions with other atoms

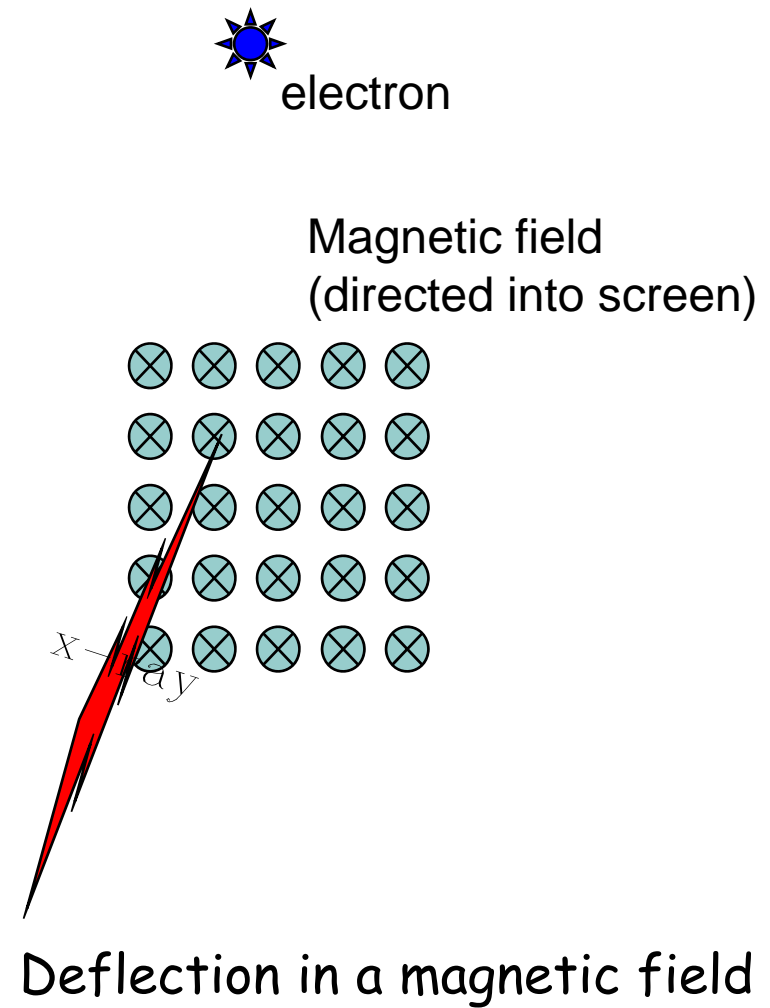
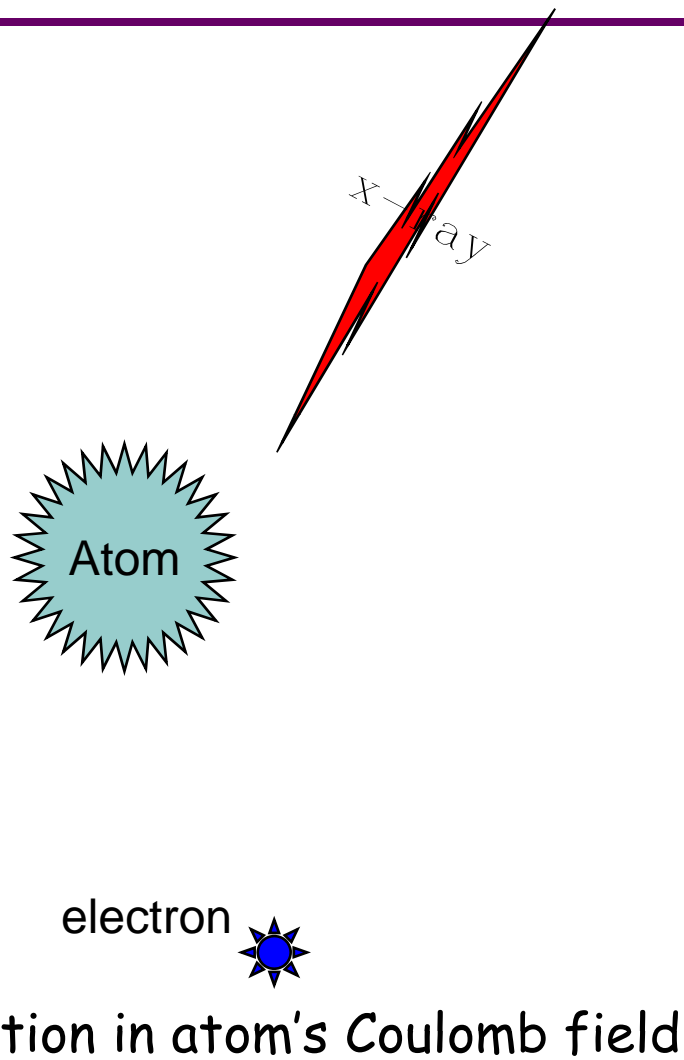
More facts about x-rays

- X-rays have energies ranging from ~ 0.5 keV up to many millions of electron-volts (MeV)
 - Radiation dose
 - Amount of energy deposited in matter by an x-ray
 - Dose = energy deposited per unit mass of material
 - Units: 1 gray (Gy) = 1 J / kg
- Two types of x-rays
 - Bremsstrahlung – “braking radiation”
 - Produced when energetic electrons are forced to liberate energy during a collision
 - Characteristic x-rays
 - Emitted by electrons to move between atomic orbits

Bremsstrahlung production

- An electron traveling at high speed can be deflected ...
 - ...by the Coulomb forces from nearby (heavy) atoms
 - The principle used by medical x-ray sources
 - ...by strong electromagnetic fields
 - The principle used by synchrotrons
- In either case, conservation of energy requires the electron to emit energy as its direction of propagation changes
 - The energy of the emitted x-ray depends on the magnitude of the direction change

Bremsstrahlung production



Synchrotron light source

- A synchrotron is a device for producing light by bending the paths of high-speed electrons
 - Synchrotrons are electron-storage rings
 - They “recharge” the electrons after each bend
 - Using electrical fields to re-accelerate the electrons to replace the energy lost as light
- The CAMD facility is a synchrotron
 - Using 1.3 GeV electrons
 - To produce broad-spectrum light
 - From visible light up to x-rays
 - Other CAMD personnel are more qualified to explain synchrotrons than I am



Synchrotron x-ray features

- CAMD produces x-rays at ~ 10 keV to ~ 40 keV that can be used for x-ray tomography
 - The beam is broad-spectrum and very intense
- A particular x-ray energy can be selected from the x-ray spectrum with a monochromator
 - X-rays that don't have the desired "monochromatic" energy are removed from the beam
- The CAMD x-ray beam is highly collimated
 - The x-rays are traveling essentially on parallel paths
- But the x-ray beam size is small
 - ~ 1 mm high x 30 mm wide

Synchrotron tomography

- These features of the CAMD x-ray beam...
 - Monochromatic x-rays, parallel beam, high intensity
- ...result in favorable properties for tomography imaging...
 - High resolution
 - the ability to see very small structures
 - High contrast
 - The ability to distinguish relatively similar objects
 - Potential for quantitative analysis
- ...but only for very small objects
 - Because of the small beam size

X-ray imaging and tomography

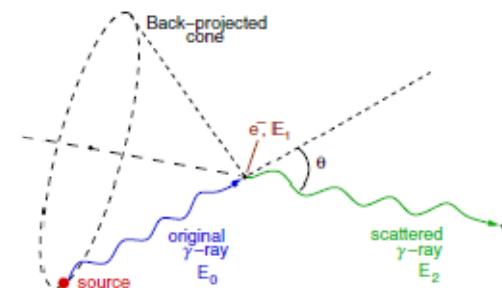
- So, CAMD can be used for tomography, but what is tomography?
 - For that matter, what can we measure with x-rays?
- In the next slides, we'll discuss
 - Interactions of x-rays with matter
 - Attenuation of x-rays in matter
 - Using beams of x-rays to measure attenuation

Radiation interactions

- Interactions are the fundamental manner in which radiation transfers energy to matter
 - Radiation interacts with matter in several different physical ways, depending on
 - Radiation type and energy
 - Density and atomic number of the matter
 - Each physical mechanism of interaction has a probability that it will occur
- For x-rays in the energy range that is useful for tomography, the primary interactions are
 - Elastic and inelastic scattering
 - Photoelectric absorption

X-ray interactions

- Photoelectric absorption
 - An x-ray interacts with an inner-shell orbital electron
 - The x-ray transfers all of its energy to the electron and ceases to exist
 - The x-ray energy must exceed the orbital binding energy for photoelectric absorption to occur
- Compton (inelastic) scattering
 - An x-ray interacts with an outer-shell orbital electron
 - The x-ray transfers some energy to the electron
 - Having lost energy, the x-ray is forced to depart in a new direction with less energy

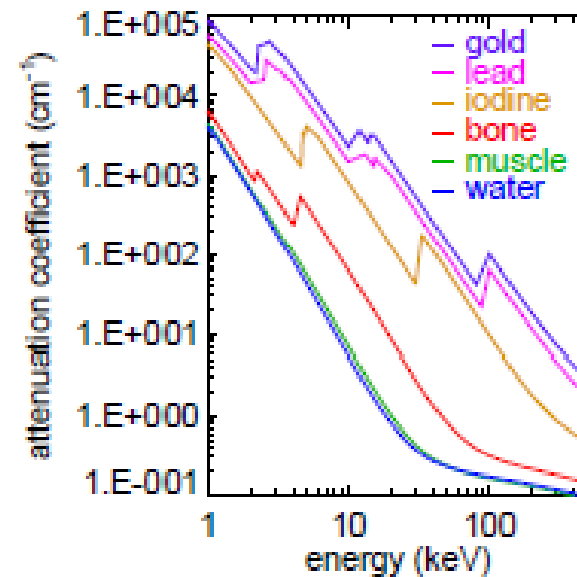
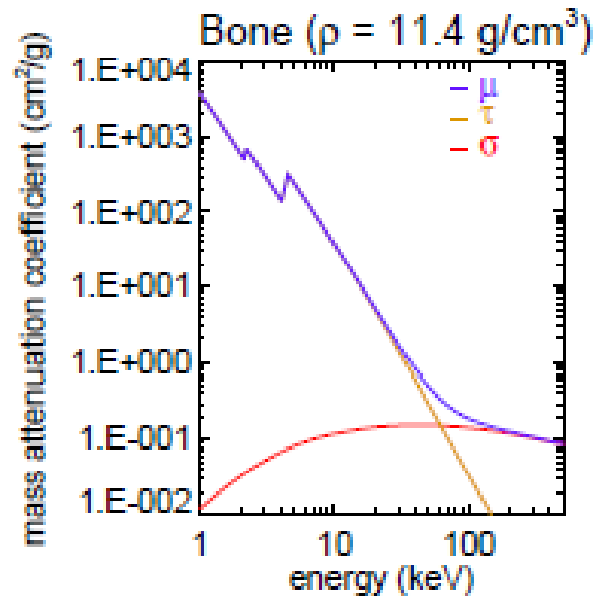


X-ray interactions

- For a beam of x-rays, both types of interactions remove x-rays from the beam
 - Photoelectric absorption destroys the x-ray
 - Compton (inelastic) scattering sends the x-ray in a different direction
- Both types of interactions have a likelihood that they will occur
 - τ = probability of photoelectric absorption
 - σ = probability of Compton scattering
- Total probability of interaction is $\mu = \tau + \sigma$

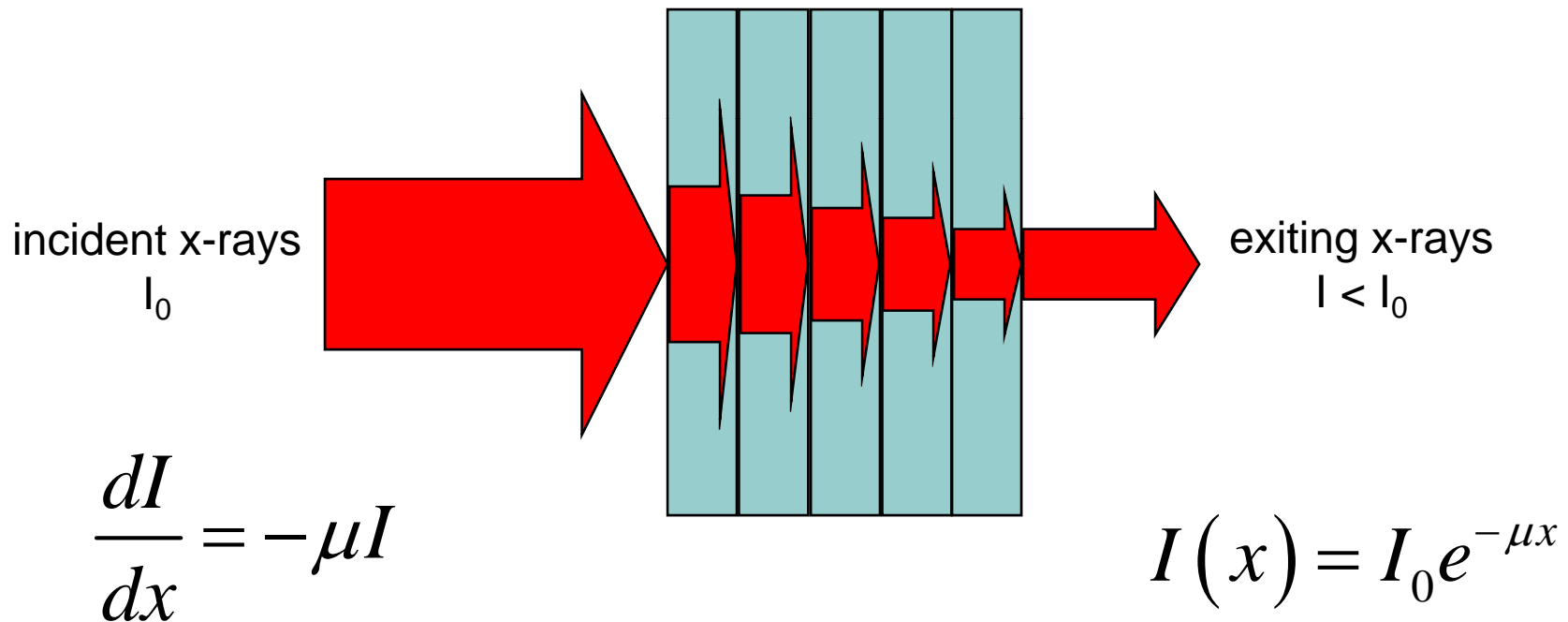
Attenuation coefficients

- The magnitude of τ and σ (and hence μ) depends on
 - The x-ray energy
 - The type of matter
 - Especially, density ρ and atomic number Z



Attenuation of x-rays in matter

- The interaction of x-rays in matter is a statistical (stochastic) process

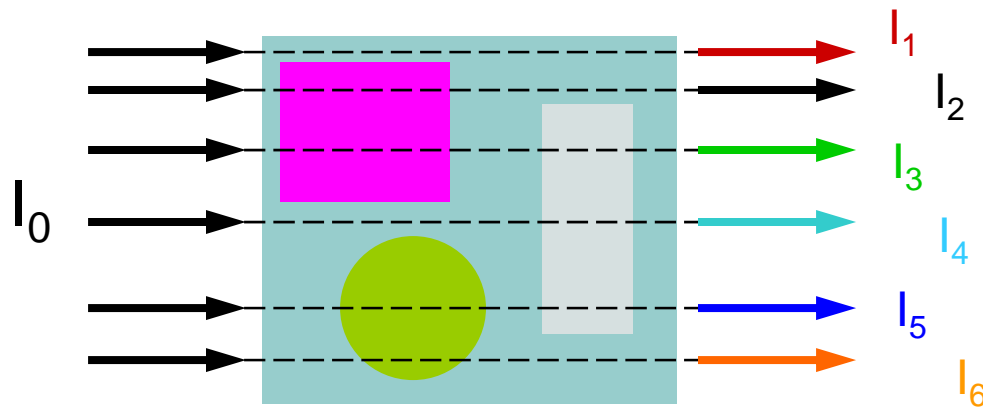


- If we know the initial intensity I_0 , and we measure $I(x)$ for a thickness x of material, we can calculate μ

Measuring attenuation with x-rays

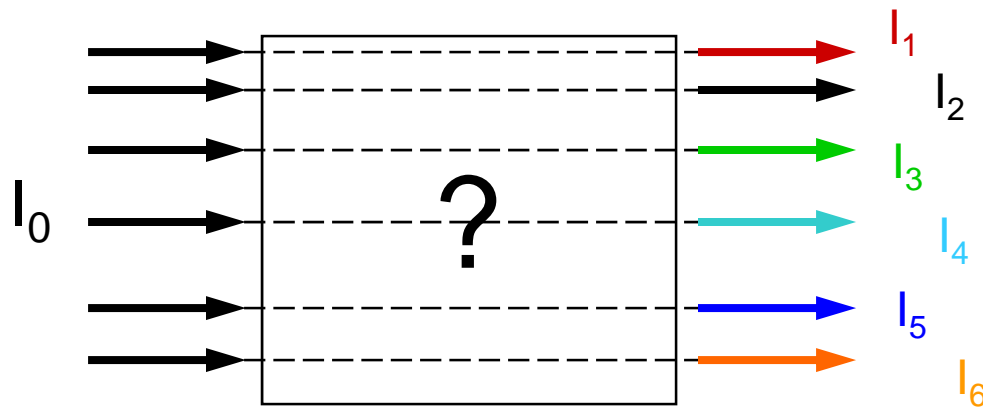
- Usually, we don't have a simple homogeneous object of constant thickness
 - we want to learn about μ as a function of location in the object
 - Fortunately, x-rays can do calculus...

$$I(x', y') = I_0 e^{-\int \mu(x, y, z; Z, \rho, E) dl}$$



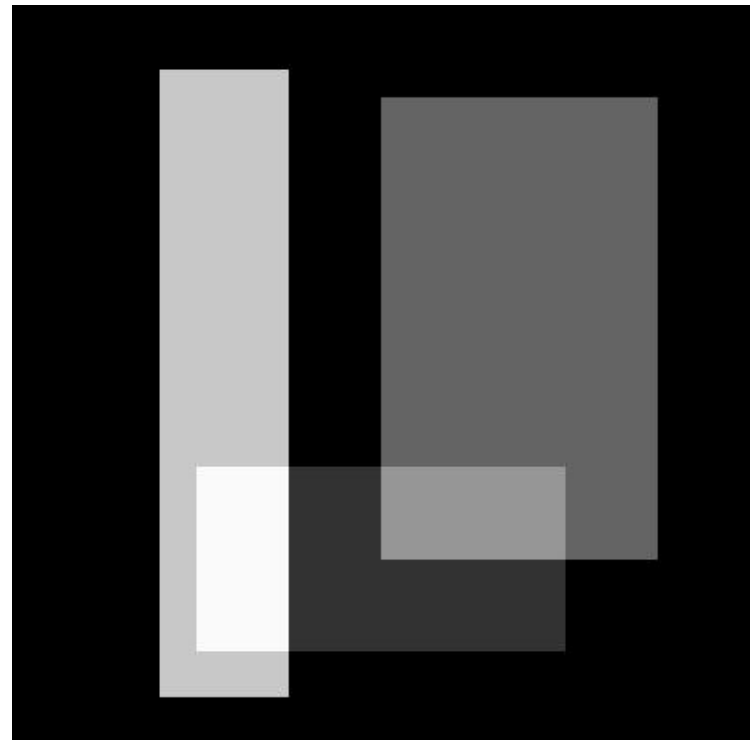
Spatial measurement of attenuation

- But this is only measuring total attenuation along the paths of the x-rays
 - We see how attenuation varies side-to-side
 - but we can't see how attenuation varies with depth in the object
 - i.e., along the paths of length L
 - The x-rays only report the integral (summed) attenuation



Superposition of structures

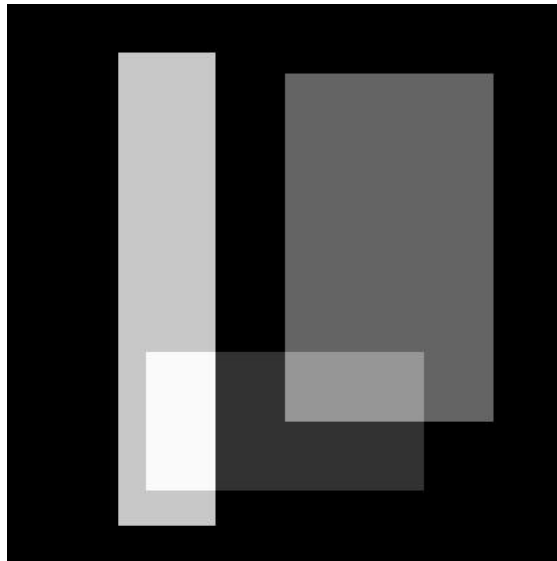
- For a 3D object, the x-rays give us a 2D picture of summed attenuation
 - We lose all “depth perception” because the x-rays have integrated along the paths they traveled
 - Each 2D picture is a projection of the 3D object
- Example: an object comprising three boxes
 - how deep is each box?
 - which box is closest? which is furthest?



Superposition of structures

- No matter which way you send the x-rays through the object, some superposition of internal structures will still occur
- But... notice that the appearance DOES change with a different perspective

front
view



side
view



The basis for tomography

- Although each perspective gives only a 2D picture of (summed) attenuation through the object along that path...
 - and each picture suffers from some superposition
- ...each perspective provides somewhat different (yet related) information about the relative locations, thicknesses, and local attenuation coefficients of the internal structures

$$I(x', y') = I_0 e^{-\int_L \mu(x, y, z; Z, \rho, E) dl}$$

- each image is summed over a different set of L's

The basis for tomography

- Thus, each image $I(x',y')$ is one set of summations of $\mu(x,y,z)$
 - For instance, one image sums by sending the x-rays parallel to the z-axis
 - but another image views the object from the side, sending the x-rays parallel to the y-axis
- If each (x,y,z) location in the object is a variable
 - whose value is the attenuation coefficient μ
 - then all of the projections together form a system of linear equations in terms of μ at each (x,y,z)
- By solving this system of linear equations, we can find $\mu(x,y,z)$ from all the $\{I(x',y')\}$ images

Solving for $\mu(x,y,z)$

- In practice, this is a huge system of linear equations
 - on the order of 1-million variables
 - each “pixel” in the tomography image is one variable
 - so you need 1-million measurements (sort of...)
- This can't be solved algebraically, even with modern array-processor computers
 - because of reasons related to
 - computation time
 - computational instability due to measurement errors

Math for tomography

- 1917: Austrian mathematician Johann Radon derived a calculus-based method for describing the creation of the projection images from an object and subsequently recovering the object from the projections
 - 1960s: Cormack (re-)discovered the method
 - At the same time, Hounsfield was developing the first tomography hardware
 - Friday's Workshop lecture will describe hardware
 - Fun fact: the Beatles recording company EMI sponsored the development of the first tomography system
 - Cormack and Hounsfield shared a Nobel Prize for the development of tomography

The Radon Transform

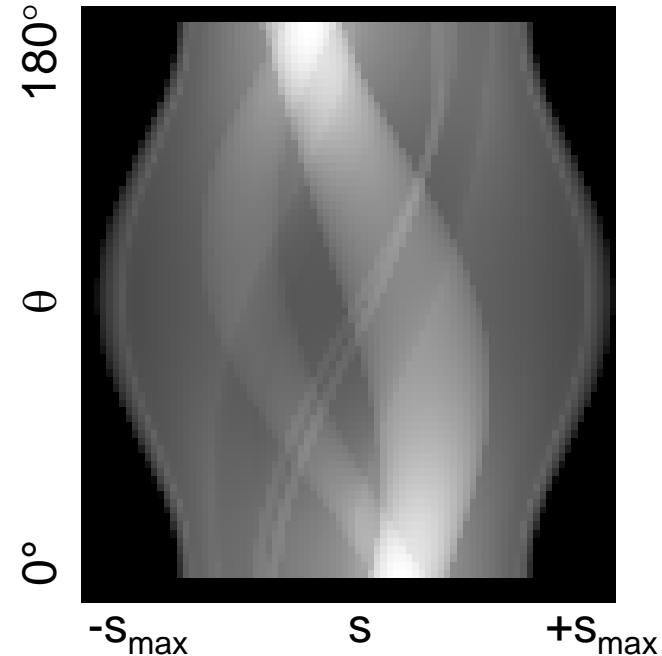
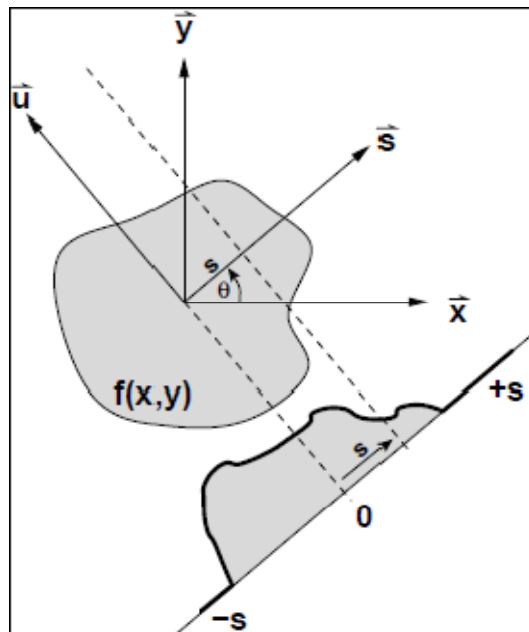
- The Radon Transform

$$g(s, \theta) = \ln \left(\frac{I_0}{I(x, y)} \right) \square \mathfrak{R}f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - s) dx dy,$$

$$-\infty < s < \infty, 0 \leq \theta < \pi$$

- $g(s, \theta)$ is the sinogram of the object

geometry
for
tomography




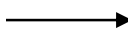

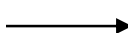






But what does the object look like?

- We must reconstruct the image from the sinogram
 - Mathematically, this is solving the large set of simultaneous equations
 - or applying the Inverse Radon Transform

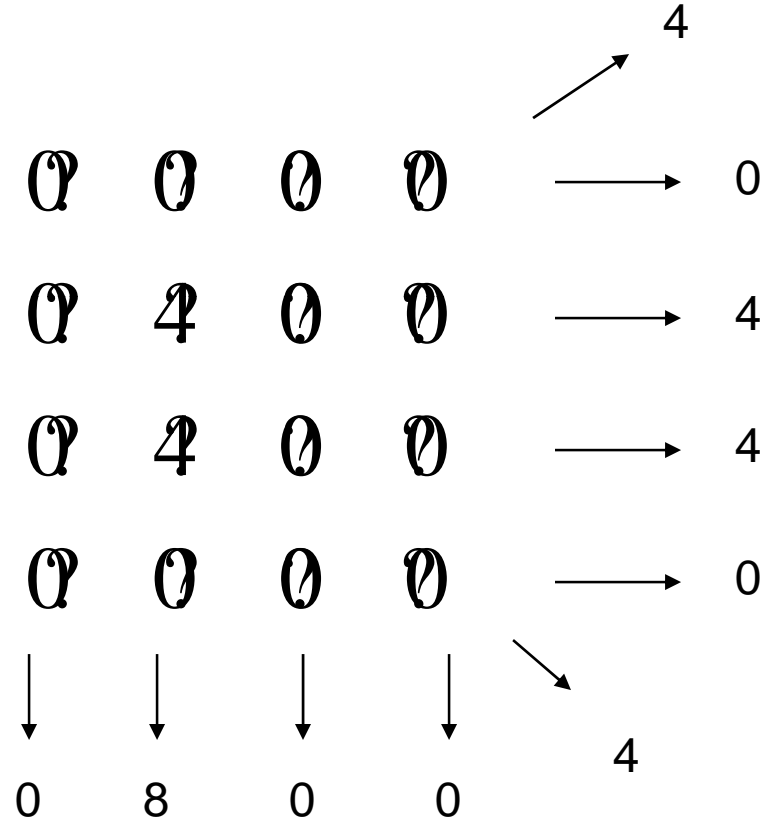
$$f(x, y) = \frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^{\infty} \frac{[(\partial g / \partial s)(s, \theta)]}{x \cos \theta + y \sin \theta - s} ds d\theta$$

- Although an elegant expression, this equation is difficult to implement in a computer
- A computationally tractable variant is called “filtered backprojection”

Magic square approach (projections)

					1
1	1	1	1		4
0	2	0	2		4
0	0	10	0		10
0	1	0	0		1
					
1	4	11	3		13

Magic square approach (reconstruction)

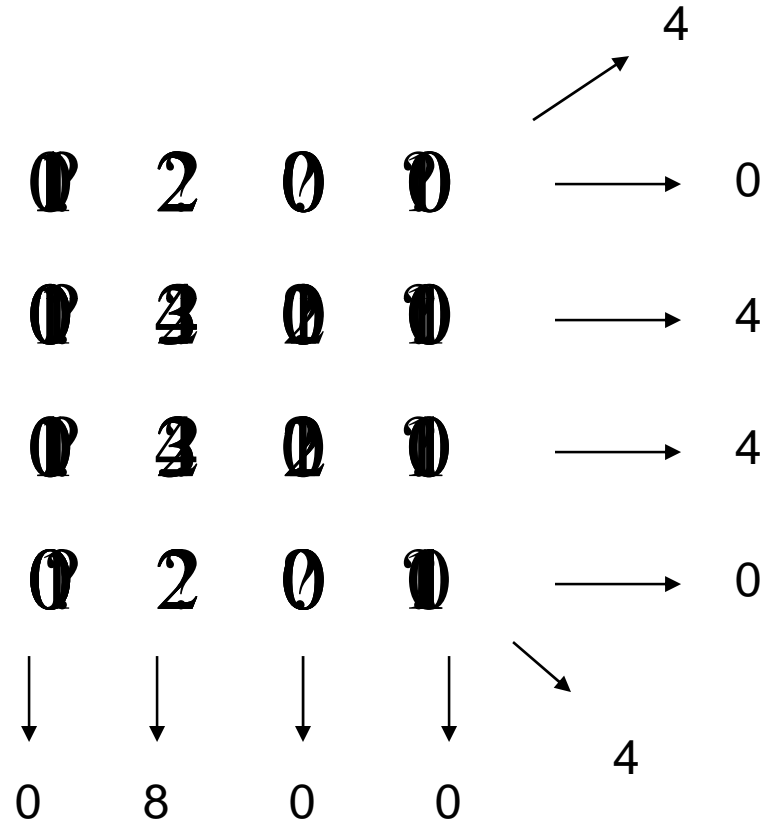


Backprojection

- The basic concept for finding the unknown values is called backprojection
 - We know that the unknown values along a line must add up to the (measured) total
 - but we don't know the magnitude of each value.
 - In practice, we assume each pixel (value) contributes equally to the total
 - For each line, assign a fraction of the total to each pixel along the line



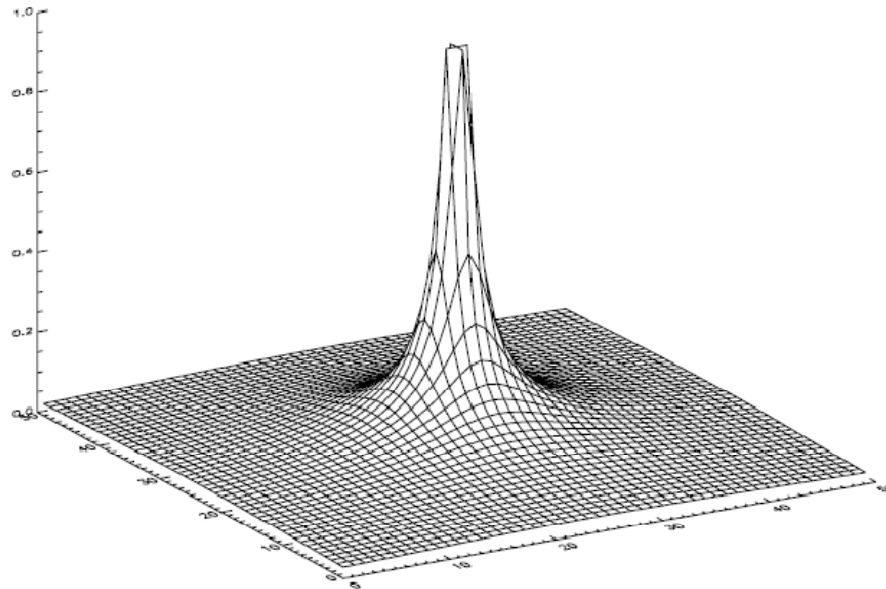
Magic square approach (backprojection)



We see the values we want (the 4s), but there are surrounding “garbage”

1/r blurring artifact

- The extra “garbage” values are an artifact of our assumption that each pixel contributed equally to the total along each line
 - each pixel in the image bleeds its value out into its surroundings

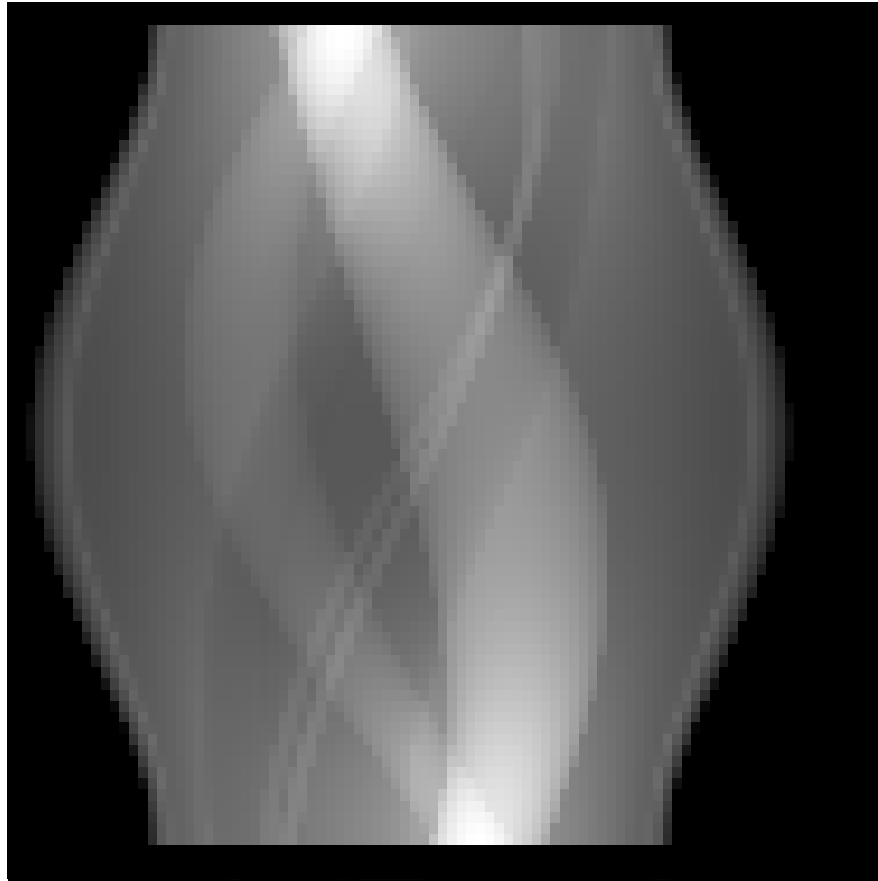


Filtered backprojection

- The solution to the blurring artifact is to apply an image processing filter
 - The filter “cancels” the blurring artifact
 - The filter is called the Ramp filter because of its shape in Fourier space
- How the Ramp filter works
 - Effectively, it adds small negative values adjacent to the line that is being backprojected
 - These negative values adjacent to each line add together in just the right amount to cancel out all the garbage values

Filtered backprojection

Reconstructing...



Sinogram

Reconstructed
image

Advanced tomography topics

- Tomography in the real world
 - Sampling and incomplete data
 - Discrete tomography
 - Iterative reconstruction methods
 - Quantitative image analysis methods
 - and many many more topics

Resources

- Barrett, H and Myers, K. **Foundations of Image Science**. Wiley-Interscience: Hoboken, NJ, 2004
- Deans, S. **The Radon Transform and Some of Its Applications**. Krieger Publishing Co.: Malabar, FL, 1993
- Jain, A. **Fundamentals of Digital Image Processing**. Prentice Hall: Englewood Cliffs, 1989
- Webb, S. **The Physics of Medical Imaging**. Institute of Physics Publishing: Bristol, U.K., 1988
- Webb, S. **From the Watching of Shadows: The Origins of Radiological Tomography**. Adam Hilger/IOP Publishing: Bristol, U.K., 1990

